

Differentiability at the origin

Given

$$z = \sqrt{x^4 + y}$$

is it differentiable at the origin?

Solution

To be differentiable at the origin, the partial derivatives at the origin must exist. But the partial derivatives do not exist at the origin. Therefore, even though the function is continuous at the origin, it is not differentiable.

$$z'_x = \frac{x^3}{(x^4 + y)^{3/4}}$$

$$z'_y = \frac{1}{(x^4 + y)^{3/4}}$$